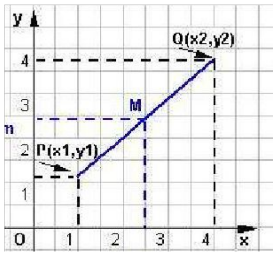
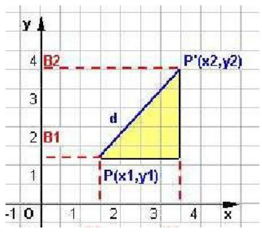
FORMULARIO DI GEOMETRIA ANALITICA

Punto medio tra due punti.



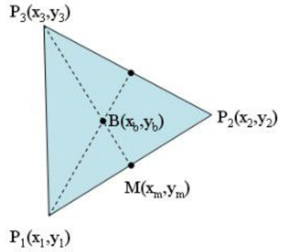
$$x_M=\frac{x_1+x_2}{2} \quad , \quad y_M=\frac{y_1+y_2}{2}$$

Distanza fra due punti.



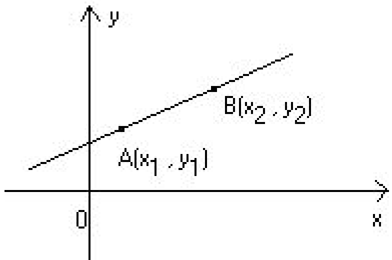
$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Baricentro di un triangolo.



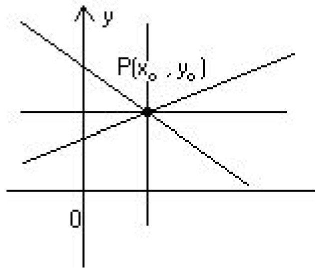
$$x_0=\frac{x_1+x_2+x_3}{3} \quad , \quad y_0=\frac{y_1+y_2+y_3}{3}$$

Retta per due punti.



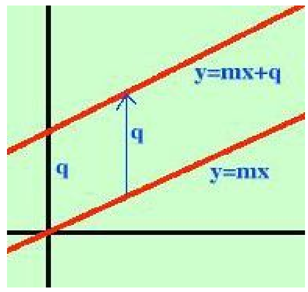
$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$$

Retta per un punto.



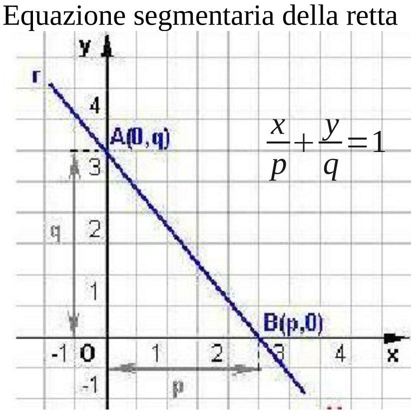
$$y-y_0=m\cdot(x-x_0)$$

Equazione esplicita della retta.



$$m=\tan\alpha \qquad y=m\cdot x+q$$

Equazione segmentaria della retta



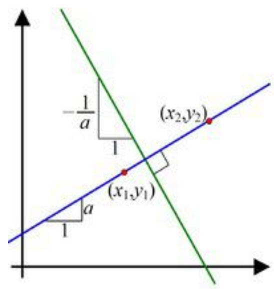
$$\frac{x}{p}+\frac{y}{q}=1$$

Equazione implicita della retta.

$$a\cdot x+b\cdot y+c=0$$

$$m=-\frac{a}{b} \qquad q=-\frac{c}{b}$$

Rette perpendicolari.



$$m'=-\frac{1}{m}$$

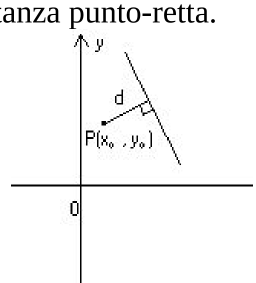
Posizione reciproca di due rette

$$\begin{cases} ax+by+c=0 \\ a'x+b'y+c'=0 \end{cases}$$

Sistema:

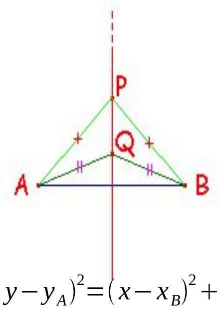
- indeterminato → rette coincidenti
- impossibile → rette paralele e distinte
- determinato → rette incidenti

Distanza punto-retta.



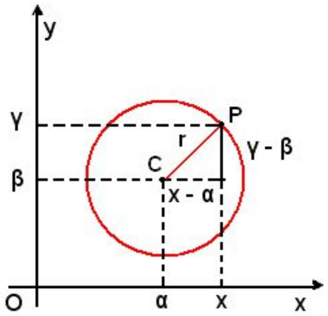
$$d=\frac{|ax_0+bx_0+c|}{\sqrt{a^2+b^2}}$$

Asse di un segmento.



$$(x-x_A)^2+(y-y_A)^2=(x-x_B)^2+(y-y_B)^2$$

Equazione cartesiana della circonferenza.



$$(x-\alpha)^2+(y-\beta)^2=r^2$$

Equazione implicita della circonferenza.

$$x^2+y^2+ax+by+c=0$$

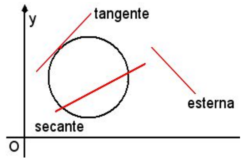
$$a=-2\alpha \quad b=-2\beta$$

$$c=\ \alpha^2+\beta^2-r^2$$

Circonferenza e retta.

Risolvere il sistema:

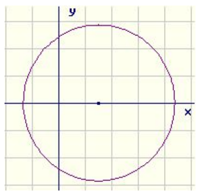
$$\begin{cases} ax+by+c=0 \\ a'x+b'y+c'=0 \end{cases}$$



SE:

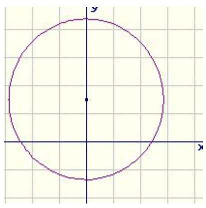
- $\Delta<0$ → esterna
- $\Delta=0$ → tangente
- $\Delta>0$ → secante

Con centro sull'asse x



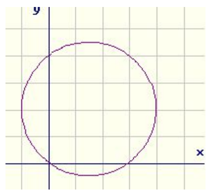
$$x^2+y^2+ax+c=0$$

Con centro sull'asse y



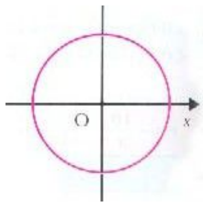
$$x^2+y^2+by+c=0$$

Passante per l' origine



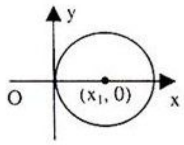
$$x^2+y^2+ax+by=0$$

Centro sull'origine



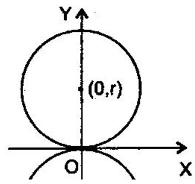
$$x^2+y^2+c=0$$

Centro asse x e passante origine



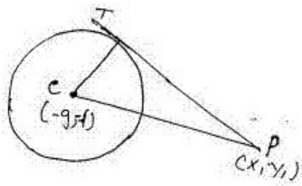
$$x^2+y^2+ax=0$$

Centro asse y e passante origine



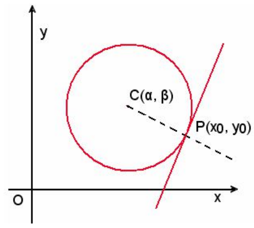
$$x^2+y^2+bx=0$$

Retta tangente.



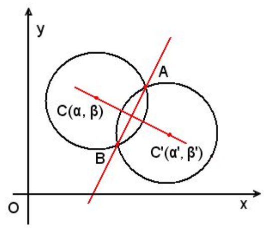
$$r=\frac{|m\alpha-\beta+y_1-mx_1|}{\sqrt{1+m^2}}$$

Regola dello sdoppiamento:



$$x\cdot x_0+y\cdot y_0+a\frac{x+x_0}{2}+b\frac{y+y_0}{2}+c=0$$

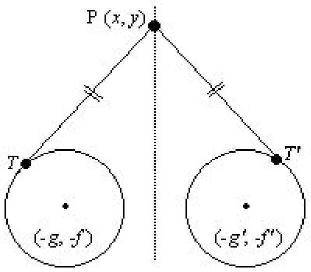
Circonferenze secanti.



Asse radicale:

$$(a-a')x+(b-b')y+(c-c')=0$$

Proprietà asse radicale

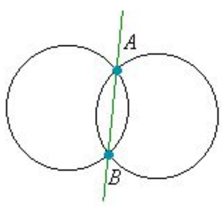


PT = PT'

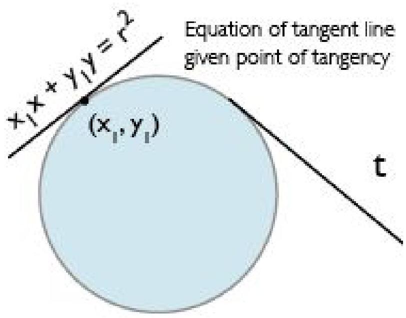
Punti intersezione.

Risolvere il sistema:

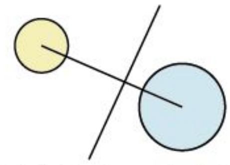
$$\begin{cases} x^2+y^2+ax+by+c=0 \\ (a-a')x+(b-b')y+(c-c')=0 \end{cases}$$



Regola dello sdoppiamento



Proprietà asse radicale:



Radical axis of non intersecting circles.

E' ortogonale alla retta per i centri (asse centrale)

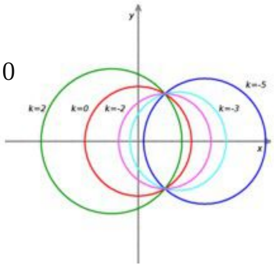
Fasci di circonferenze:

$$x^2+y^2+ax+by+c+k(x^2+y^2+a'x+b'y+c')=0$$

$$x^2+y^2+\frac{a+ka'}{k+1}x+\frac{b+kb'}{k+1}y+\frac{c+kc'}{k+1}=0$$

Asse radicale:

$$(a-a')x+(b-b')y+(c-c')=0$$

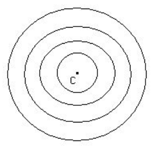


Punti base.

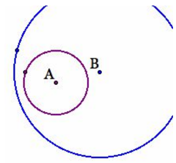
Risolvere il sistema formato da una generatrice e dall'asse radicale.

$$\left\{\begin{array}{c} x^2+y^2+ax+by+c=0 \\ (a-a')x+(b-b')y+(c-c')=0 \end{array}\right\}$$

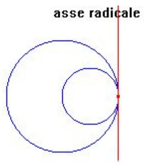
Se a=a' e b= b' → fascio concentrico



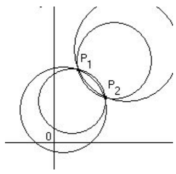
Se: Δ<0 → esterne



Δ=0 → tangenti



Δ>0 → secanti

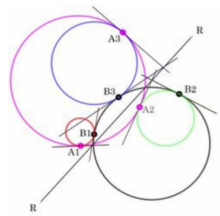


Fasci di circonferenze tangenti in un punto (x_0,y_0):

$$(x-x_0)^2+(y-y_0)^2+k(ax+by+c)=0$$

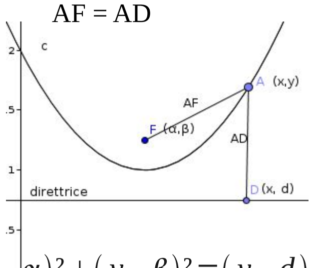
$$a\cdot x+b\cdot y+c=0$$

è l'equazione dell'asse radicale



Equazione cartesiana della parabola.

AF = AD



$$(x-\alpha)^2+(y-\beta)^2=(y-d)^2$$

$y=ax^2+bx+c$ asse simmetria // asse y

$x=ay^2+by+c$ asse simmetria // asse x

Caratteristiche di y=ax²+bx+c

$$\Delta=b^2-4ac$$

Fuoco e vertice:

$$F\left(\frac{-b}{2a},\frac{1-\Delta}{4a}\right) \quad V\left(\frac{-b}{2a},\frac{-\Delta}{4a}\right)$$

Direttrice e asse di simmetria:

$$y_d=-\frac{1+\Delta}{4a} \quad x_S=\frac{-b}{2a}$$

Coefficienti da parametri geometrici:

$$a=\frac{1}{2(\beta-d)} \quad b=\frac{-\alpha}{\beta-d} \quad c=\frac{\alpha^2+\beta^2-d^2}{2(\beta-d)}$$

Caratteristiche di x=ay²+by+c

$$\Delta=b^2-4ac$$

Fuoco e vertice:

$$F\left(\frac{1-\Delta}{4a},\frac{-b}{2a}\right) \quad V\left(\frac{-\Delta}{4a},\frac{-b}{2a}\right)$$

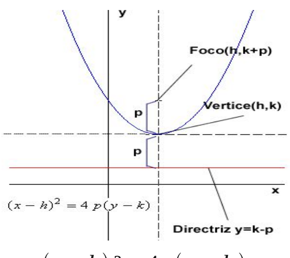
Direttrice e asse di simmetria:

$$x_d=-\frac{1+\Delta}{4a} \quad y_S=\frac{-b}{2a}$$

Coefficienti da parametri geometrici:

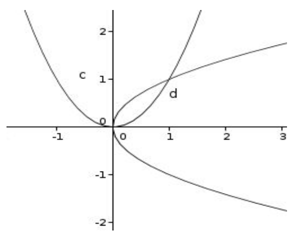
$$a=\frac{1}{2(\alpha-d)} \quad b=\frac{-\beta}{\alpha-d} \quad c=\frac{\alpha^2+\beta^2-d^2}{2(\alpha-d)}$$

Altra equazione della parabola

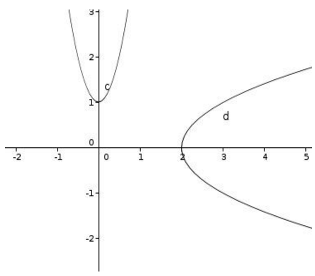


$$(x-h)^2=4p(y-h)$$

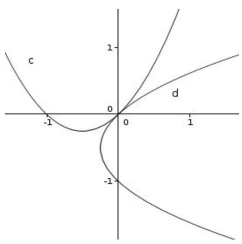
Particolari parabole (b = 0 e c= 0)



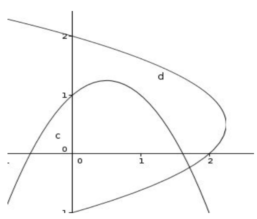
Particolari parabole (b= 0)



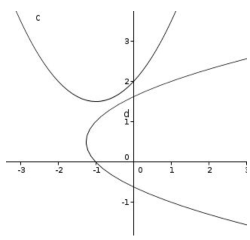
Particolari parabole (c= 0)



Parabole con a < 0:

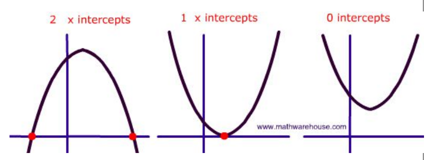


Parabole con a > 0



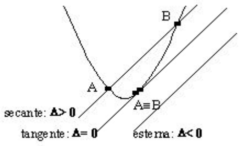
Intersezione con l'asse x (radici)

Risolvere: $0=ax^2+bx+c$



Parabola e retta

Risolvere: $ax^2+(b-m)x+c-q=0$



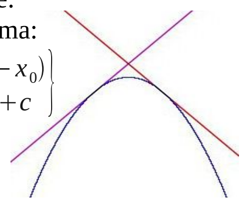
SE:

- $\Delta<0 \rightarrow$ esterna
- $\Delta=0 \rightarrow$ tangente
- $\Delta>0 \rightarrow$ secante

Rette tangenti a parabola da un punto non appartenente:

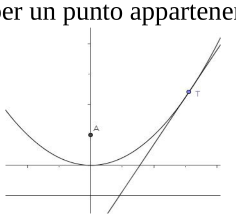
Generare il sistema:

$$\begin{cases} y-y_0=m(x-x_0) \\ y=ax^2+bx+c \end{cases}$$



Dal sistema risolvere $\Delta(m)=0$

Tangente per un punto appartenente



$m=2ax_0+b$

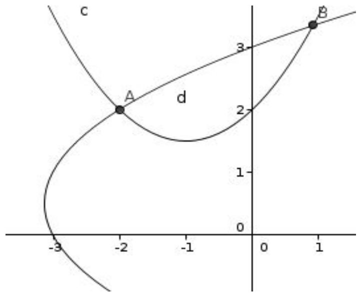
$$\frac{y+y_0}{2}=ax_0\,x++b\frac{x+x_0}{2}+c$$

$m=1/(2ay_0+b)$

$$\frac{x+x_0}{2}=ay_0\,y++b\frac{y+y_0}{2}+c$$

Parabole secanti.

Risolvere il sistema formato dalle equazioni delle due parabole.



Fasci di parabole.

$$y-ax^2-bx-c+k(y-a'x^2-b'x-c')=0$$

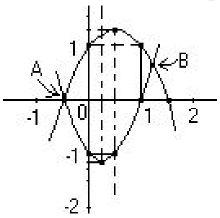
$$y=\frac{a+a'k}{1+k}x^2+\frac{b+b'k}{1+k}x+\frac{c+c'k}{1+k}$$

<u>Punti base</u>: risolvere il sistema di secondo grado delle due generatrici

SE:

- $\Delta<0 \rightarrow$ senza punti base
- $\Delta=0 \rightarrow$ un punto base
- $\Delta>0 \rightarrow$ due punti base

Dati i punti base $A(x_A,y_A)$ e $B(x_B,y_B)$ trovare il fascio di parabole.



Trovare a retta:

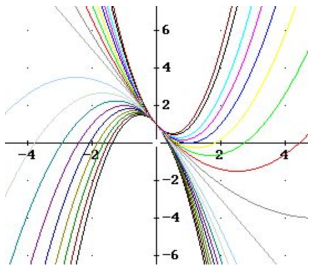
$$\frac{x-x_A}{x_B-x_A}=\frac{y-y_A}{y_B-y_A}$$

e poi:

$$y=m\cdot x+q+k(x-x_A)(x-x_B)$$

è l'equazione del fascio

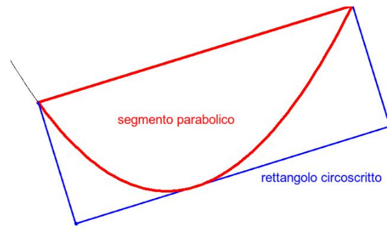
Fasci di parabole tangenti.



Asse radicale : $y - y_T = m(x - x_T)$

Fascio: $y= y_T+m(x-x_T) + k\cdot(x-x_T)^2$

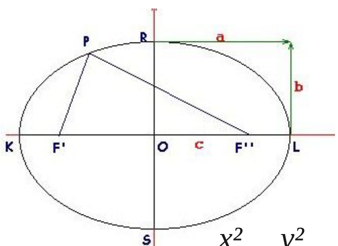
Teorema di Archimede



Area del segmento parabolico = 2·(Area del rettangolo circoscritto) /3

Equazione canonica dell'ellisse.

Se $a^2-b^2=c^2>0 \rightarrow$ fuochi su asse x



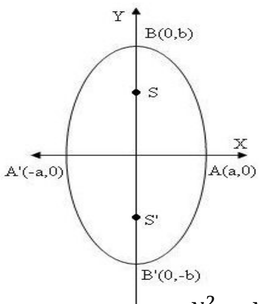
Equazione: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Fuochi: $F_1(-c,0)$ $F_2(c,0)$

Eccentricità: e= c/a < 1

Equazione canonica dell'ellisse.

Se $b^2-a^2=c^2>0 \rightarrow$ fuochi su asse y



Equazione: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Fuochi: $F_1(0,c)$ $F_2(0,-c)$

Eccentricità: e= c/b < 1

Ellisse e retta.

Risolvere il sistema:

$$\begin{cases} y=mx+q \\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \end{cases}$$

SE:

- $\Delta<0 \rightarrow$ retta esterna
- $\Delta=0 \rightarrow$ retta tangente
- $\Delta>0 \rightarrow$ retta secante

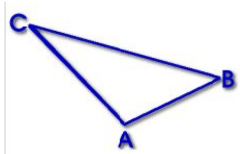
Formula di sdoppiamento:

$$\frac{xx_0}{a^2}+\frac{yy_0}{b^2}=1$$

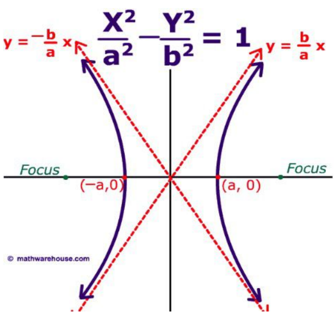
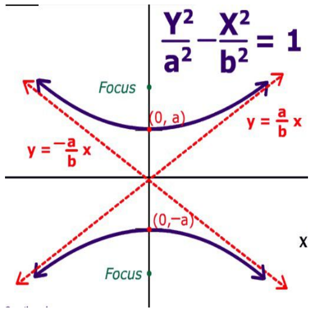
AREA DELL'ELLISSE: S = πa·b

Area di un triangolo:

$$S=\frac{1}{2}\left|y_A(x_B-x_C)+y_C(x_A-x_B)+y_B(x_C-x_A)\right|$$



Allineamento di tre punti: $\frac{x_3-x_1}{x_2-x_1}=\frac{y_3-y_1}{y_2-y_1}$ Distanza tra due rette parallele: $d=\frac{|q-q'|}{\sqrt{1+m^2}}$

Equazione canonica dell'iperbole. Condizione: $a>b$ (fuochi su asse x)  Fuochi: $c=\pm\sqrt{a^2+b^2}$ Eccentricità: e= c/a > 1 Asintoti: $y=\pm\frac{b}{a}x$	Equazione canonica dell'iperbole Condizione: $a>b$ (fuochi su asse y)  Fuochi: $c=\pm\sqrt{a^2+b^2}$ Eccentricità: e= c/a > 1 Asintoti: $y=\pm\frac{a}{b}x$	Iperbole e retta. Risolvere il sistema: $\begin{cases} y=mx+q \\ \frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \end{cases}$ SE: • Δ<0 → retta esterna • Δ=0 → retta tangente • Δ>0 → retta secante Formula di sdoppiamento: $\frac{xx_0}{a^2}-\frac{yy_0}{b^2}=1$ oppure $\frac{yy_0}{a^2}-\frac{xx_0}{b^2}=1$